

# Gap Symmetry and Thermal Conductivity in Nodal Superconductors

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## Abstract

Here we consider the universal heat conduction and the angular dependent thermal conductivity in the vortex state for a few nodal superconductors. We present the thermal conductivity as a function of impurity concentration and the angular dependent thermal conductivity in a few nodal superconductors. This provides further insight in the gap symmetry of superconductivity in  $\text{Sr}_2\text{RuO}_4$  and  $\text{UPd}_2\text{Al}_3$ .

*Key words:* Gap symmetry, thermal conductivity, nodal superconductors, impurity scattering.

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Since the appearance of heavy fermion superconductors, organic superconductors, high  $T_c$  cuprate superconductors,  $\text{Sr}_2\text{RuO}_4$ , the gap symmetry has been the central issue for these new superconductors. In the last few years, Izawa *et al.* have succeeded in identifying the energy gap in  $\text{Sr}_2\text{RuO}_4$ ,  $\text{CeCoIn}_5$ ,  $\kappa\text{-(ET)}_2\text{Cu(NCS)}_2$ ,  $\text{YNi}_2\text{B}_2\text{C}$  and  $\text{PrOs}_4\text{Sb}_{12}$  [1,2,3,4,5]. In these works the angular dependent thermal conductivity in the vortex state of nodal superconductors has played the crucial role [6,7]. Here we show two new results: the universal heat conduction for different nodal superconductors and the angular dependent thermal conductivity for nodal superconductors with horizontal nodes. Until now the universal heat conduction is discussed only for d-wave superconductors in quasi 2D systems (i.e.  $\Delta(\mathbf{k}) = \Delta f$  with  $f = \cos(2\phi)$  and  $\sin(2\phi)$ ) [7,8]. Here we consider in addition to  $f$  for d-wave superconductors  $f = \sin(\chi)$ ,  $\cos(\chi)$ ,  $\cos(2\chi)$ ,  $e^{\pm i\phi} \cos(\chi)$ , and  $e^{\pm i\phi} \sin \chi$  where  $\chi = ck_z$ . Note all these states have the

same quasiparticle density of states, the gap equations and the thermodynamics[9]. Then we obtain

$$\frac{\kappa_{xx}}{\kappa_n} = \frac{2}{\pi} \frac{\Gamma_0}{\Delta} \frac{1}{\sqrt{1+C_0^2}} E\left(\frac{1}{\sqrt{1+C_0^2}}\right) \equiv I_1\left(\frac{\Gamma}{\Gamma_0}\right) \quad (1)$$

where  $\Gamma_0 \simeq 0.866T_c$ , the quasiparticle scattering rate at which the superconductivity disappears [8] and  $C_0$  is determined by

$$\frac{C_0^2}{\sqrt{1+C_0^2}} K\left(\frac{1}{\sqrt{1+C_0^2}}\right) = \frac{\pi}{2} \frac{\Gamma}{\Delta} \quad (2)$$

and  $K(z)$  and  $E(z)$  are the complete elliptic integrals. Here we assume that the impurity scattering is in the unitarity limit. So we see that Eq.(1) is valid for all  $f$ 's we have described above. In other words the planar thermal conductivity ( $\kappa_{xx} = \kappa_{yy}$ ) cannot discriminate different nodal superconductors in the absence of magnetic field.

On the other hand  $\kappa_{zz}$  is of more interest, we find

$$\frac{\kappa_{zz}}{\kappa_n} = I_1(\Gamma/\Gamma_0) \quad (3)$$

for  $f = \cos(2\phi)$ ,  $\sin(2\phi)$ , and  $\cos(2\chi)$ . But

$$\frac{\kappa_{zz}}{\kappa_n} = \frac{4}{\pi} \frac{\Gamma_0}{\Delta} \frac{1}{\sqrt{1+C_0^2}} \left\{ E\left(\frac{1}{\sqrt{1+C_0^2}}\right) - C_0^2 \left( \right. \right.$$

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<sup>1</sup> HW acknowledges the support from the Korean Science and Engineering Foundation(KOSEF) through the Grant No.R05-2004-000-1084

$$K\left(\frac{1}{\sqrt{1+C_0^2}}\right) - E\left(\frac{1}{\sqrt{1+C_0^2}}\right)\Big)\Big\} \equiv I_2\left(\frac{\Gamma}{\Gamma_0}\right) \quad (4)$$

for  $f = \cos(\chi)$ ,  $e^{\pm i\phi} \cos(\chi)$  and

$$\begin{aligned} \frac{\kappa_{zz}}{\kappa_n} &= \frac{2\Gamma_0}{\Delta} \frac{\Gamma}{\Delta} \left\{ 1 - E\left(\frac{1}{\sqrt{1+C_0^2}}\right) / K\left(\frac{1}{\sqrt{1+C_0^2}}\right) \right\} \\ &= I_3\left(\frac{\Gamma}{\Gamma_0}\right) \end{aligned} \quad (5)$$

for  $f = \sin \chi$  and  $e^{\pm i\phi} \sin \chi$ .  $I_1$ ,  $I_2$  and  $I_3$  versus  $\Gamma/\Gamma_0$  are shown in Fig. 1.

Also the data by Suzuki *et al.* [10] clearly favors f-wave superconductor  $f = e^{\pm i\phi} \cos(\chi)$  as in [1]. Further recent magneto thermal conductivity data for  $\kappa$  in UPd<sub>2</sub>Al<sub>3</sub> [11] appear to be more consistent with  $f = \cos 2\chi$ .

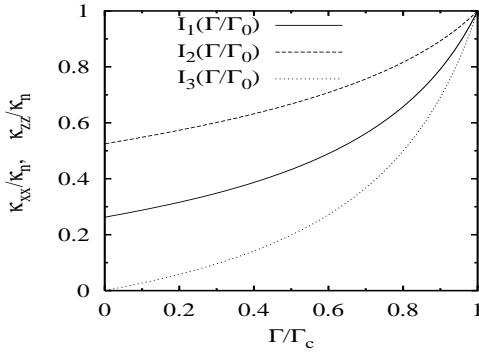


Fig. 1.  $I_i(\Gamma/\Gamma_c)$ 's in Eq.(1), Eq.(4) and Eq.(5) are shown, which represent  $\kappa_{xx}$  and  $\kappa_{zz}$  for various  $f$ 's.

The angular dependent thermal conductivity in quasi 2D nodal superconductors has been considered in [12]. However in the presence of magnetic field in the z-x plane has not been considered. Following [12] we obtain in the superclean limit ( $\sqrt{\Gamma}\Delta \ll \tilde{v}\sqrt{eH}$ )

$$\kappa_{yy}/\kappa_n = \frac{2}{\pi} \frac{\tilde{v}^2 eH}{\Delta^2} F_1(\theta) \quad (6)$$

and in the clean limit ( $\tilde{v}\sqrt{eH} \ll \sqrt{\Gamma}\Delta$ )

$$\kappa_{yy}/\kappa_{00} = 1 + \frac{\tilde{v}^2 eH}{6\pi\Gamma\Delta} F_2(\theta) \ln\left(2\sqrt{\frac{2\Delta}{\pi\Gamma}}\right) \ln\left(\frac{2\Delta}{\tilde{v}\sqrt{eH}}\right) \quad (7)$$

where

$$F_1(\theta) = \sqrt{\cos^2 \theta + \alpha \sin^2 \theta} \left( 1 + \sin^2 \theta \left( -\frac{3}{8} + \alpha \sin^2 \chi_0 \right) \right) \quad (8)$$

and

$$F_2(\theta) = \sqrt{\cos^2 \theta + \alpha \sin^2 \theta} \left( 1 + \sin^2 \theta \left( -\frac{1}{4} + \alpha \sin^2 \chi_0 \right) \right) \quad (9)$$

where  $\alpha = (v_c/v_a)^2$  and  $\chi_0$  is the nodal angle. For example  $f = \sin(\chi)$ ,  $\cos(\chi)$  and  $\cos(2\chi)$  gives  $\chi_0 = 0$ ,

$\frac{\pi}{2}$  and  $\frac{\pi}{4}$ , respectively. Here  $\kappa_n$  and  $\kappa_{00}$  are the thermal conductivity in the normal state and in the limit of  $T \rightarrow 0$  respectively. We show  $F_1(\theta)$  and  $F_2(\theta)$  for  $\chi_0 = 0$ ,  $\frac{\pi}{4}$  and  $\frac{\pi}{2}$  in Fig.2. Here we took appropriate  $\alpha = 0.69$  to UPd<sub>2</sub>Al<sub>3</sub>. Again the comparison with the

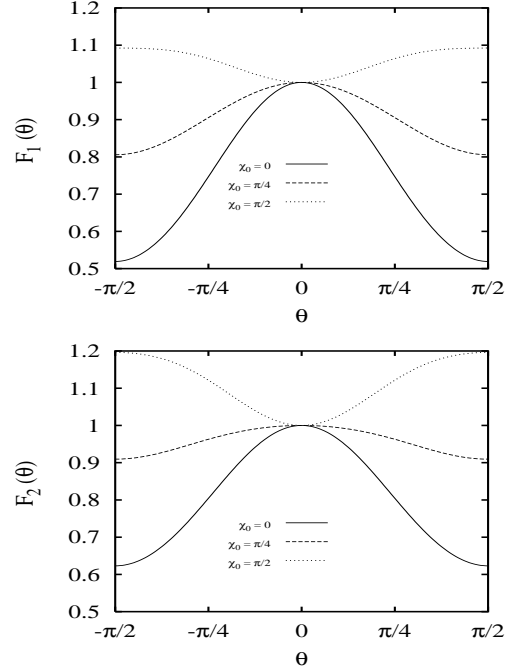


Fig. 2.  $F_1(\theta)$  in Eq.(8) and  $F_2(\theta)$  in Eq.(9) are shown, which represent the angle dependent  $\kappa_{yy}$  in the superclean limit and clean limit, respectively.

experimental data suggests for UPd<sub>2</sub>Al<sub>3</sub>. On the other hand the microscopic model  $f = \cos(\chi)$  for UPd<sub>2</sub>Al<sub>3</sub> appears to suggest  $f = \cos(\chi)$ [13]. We are benefited with useful discussions with Stephan Haas, Yuji Matsuda, Peter Thalmeier and T. Watanabe on UPd<sub>2</sub>Al<sub>3</sub>.

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